Mixed Strategies Without Mixups

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Table of Contents

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Introduction

Increasingly, introductory political science classes and textbooks are incorporating basic game theory into the curriculum (e.g. Bueno de Mesquita (2005); Clark, Golder & Golder (forthcoming); Russett, Starr, and Kinsella (2006)). Most students enrolled in these courses have had little to no exposure to basic game theory and many are shocked to find math in a political science class. This presents a pedagogical challenge to instructors: what is the most effective way to teach this material to a diverse audience of students in an engaging way?

With practice in class and through homework assignments, most students understand how to find pure strategy Nash Equilibria with relative ease. Mixed strategies, on the other hand, are less intuitively obvious to students without any background in game theory (and not always perfectly clear to those with some basic understanding of game theory). This activity is intended to present mixed strategies in an intuitively appealing way and to help the students uncover, for themselves, the logic of mixed strategies. We begin by introducing mixed strategies without any math or equations. After the students play one round of the activity, a discussion follows and only then are the equations introduced. This allows the students to gain an intuitive understanding of mixed strategies prior to seeing the mathematical solution to the game. Our target audience is an introduction to international relations class where students have little to no background in game theory, with the important exception that they have learned to solve for pure strategy Nash Equilibria. We do not necessarily expect that students will be able to solve for mixed strategy equilibria following the completion of this activity, but that they will have an understanding of (1) what a mixed strategy is compared to a pure strategy and (2) the purpose of mixed strategies and why they are preferable to pure strategies.

We recommend that instructors do a brief trial run prior to introducing this activity in class, particularly if they have little or no experience running games in class. We assume a 75-minute class session. Briefly, the students are put into pairs and play out a series of rounds. In the first round, one student randomizers her strategy by selecting a poker chip out of a opaque bag, while the other student responds by attempting to pick the best response strategy.[1] The students play a large number of rounds (approximately 15-20) so that the actual play of the randomizing student reflects the underlying probability distribution of poker chips in the bag. Following play, the class discusses the outcomes of the first round and the difficulty of the responding student to identify a "best reply" to the randomizing student's moves.[2]

Next, the instructor leads the class in solving for the mixed strategy equilibria, using only the responding student's payoffs (leaving the "randomizer" to randomize with their bags). At this point of the activity, the instructor emphasizes the key concepts underlying mixed strategies. After the class has identified the mixed strategy for the responding student, the instructor

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[1] A detailed lesson plan follows. Here, we just briefly outline the activity.
[2] Instructors or classes who have less experience with the use of classroom games to teach strategic interaction may wish to consult Powner and Croco (2005). In particular, prior playing of the Iterated Prisoner's Dilemma game there may be quite useful in acclimating students to the idea and effect of playing multiple rounds with the same payoff structure.

distributes new bags with poker chips that reflect this probability distribution. The student pairs then play Round 2, another 15-20 plays.[3] Then, the class discusses how the rounds differed and compare final totals for players. The lesson plan includes more detailed questions and prompts to direct the discussion to encourage students to identify the importance of mixed strategies and the underlying logic of them.

Instructors can apply any number of stories to fit the game presented in the activity. We provide one example, where the actors are U.S. forces and Iraqi insurgents. The U.S. forces select an area of Baghdad to patrol and protect from insurgency, while the Iraqi insurgents simultaneously select an area to attack. For the case of the model, we assume two areas: Area A which is a Shia-majority area and Area B which is Sunni-majority. The Iraqi insurgents, in this model, would most prefer to attack Area A (if the U.S. is patrolling B). Their second-preferred outcome is to attack Area B (if the U.S. is patrolling A). After that, the insurgents would prefer to attack Area A even if the U.S. is patrolling Area A (we assume there is some publicity value to Area A that the insurgents value). Finally, the least preferred outcome for the insurgents is to attack Area B while the U.S. forces are patrolling Area B. Due to the publicity value of Area A, we model that the US would most prefer to patrol A if the insurgents attack A. Following that, the U.S. forces would prefer to patrol Area B when the insurgents attack Area B. If the U.S. troops patrol the "wrong" area, they would prefer to patrol A while the insurgents attack B to patrolling B while the insurgents attack A. Of course this is a highly stylized story. The game has no pure strategy equilibria, thus fitting well into the activity introducing mixed strategies (see appendix for complete game).

Objectives

At the end of the lesson, the introductory/exposure-based student will:

- Define a mixed strategy and explain its purpose.
- Explain when and why mixed strategies are preferable to pure strategies.
- Explain why the best response to a mixed strategy is a mixed strategy.
- Have a basic exposure to the use of algebra in political science.

In addition, the modeling student will:

- Solve for mixed strategies in a 2x2 game independently using the student Section Guide handout as a guide.

Prerequisite Skills

Prior to using this lesson plan, students must:

- Be able to solve 2x2 strategic (normal)-form games; familiarity with basic games like Battle of the Sexes and Coordination is useful in discussion but not strictly necessary for the lesson.[4]
- Have a basic familiarity with the concepts of probability and probability distributions.

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[3] We also ask groups using the same strategies to average their results to increase the chance of having student play in each round approximate the underlying probability distributions.

[4] A reproducible student handout on solving normal (or strategic) form games is available on Powner's website, http://www-personal.umich.edu/~lpowner/160.html .

- Have algebra skills roughly equivalent to high school Algebra I (distributive property with variables).
- Modeling students especially should have prior exposure to the concept of indifference; if not, you may wish to insert a brief discussion around step 8 in the Procedures below.

Supplies

- Student reproducible ("Section Guide"), pp 10-11 of this packet – one copy per student[5]
- Small opaque bags (lunch bags, small fabric drawstring bags, etc.) – one per student, divided into two sets of equal numbers
- Poker chips, tokens, dried lima beans, etc. – two different colors, but items should be indistinguishable to the touch (quantity required varies)
- Instructor discussion transparencies, pages 12-13 of this packet – one copy on transparencies for overhead use (optional)
- Overhead transparency pens (optional)
- Calculator (optional)
- Masking tape or similar (optional but recommended)

Preparation

Prior to teaching, instructor should make photocopies as needed and prepare bags for students. This activity revolves around two sets of bags, each with two colors of tokens, poker chips, marked dried lima beans, etc. Pairs of students draw their moves from this bag to simulate random selection of moves; the distribution of colors represents the probability with which each move is chosen. One student in each pair is the 'randomizer,' who picks his moves randomly from the start. The second student is the 'responder,' whose distribution is the optimal mixed strategy *but* who picks randomly only in the second round of play.

Randomizer bags: Prepare at least one bag with each of the following distributions of chips or tokens.

A. 1 red, 11 white[6]
B. 11 red, 1 white
C. 5 red, 7 white
D. 7 red, 5 white
E. 3 red, 9 white (optimal strategy)

If the number of student pairs allows, we recommend two or more pairs playing strategies A and B, and two or more pairs playing strategies C and D. If desired, use masking tape or similar to tag bags with some symbol representing the distribution (ABCDE, shapes, etc). This facilitates pooling results in step 7.

Responder bags: Prepare an equal number of bags with the optimal response strategy, 6 blue, 6 green.[7]

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[5] This packet, including the student reproducible and the instructor slides, is available in MS Word format at http://www-personal.umich.edu/~lpowner for those who want to alter the pages to reflect their own course numbering, choice of example, chip-color-to-strategy labels, etc.
[6] The color choices are arbitrary but should be consistent across all randomizers' bags. Adjust student instructions below for translating colors to game moves to reflect your color choices.

Procedure

1. Distribute Section Guides to students; ask students to sit with (or facing, if possible) a partner.
2. Review story and game matrix with students. Have students verify that no solution exists in pure strategies. Review what this means: that no one move is a best response to any other move. In an introductory class, you may wish to review as well which payoff in the cell belongs to which player. Both games for Round 1 and Round 2 are duplicated on the instructor transparency (page 12 below).
3. Have student pairs assign roles in the game as Insurgents (here, randomizers) or US Forces (here, responders). Each pair should have one player in each role.
4. **Round 1.** Distribute Randomizer bags to Insurgent students. Display chips and identify which color corresponds to which game move. Explain to students that the sequence of moves for this round is:
   a) Players simultaneously select their moves. The Insurgents select a move by drawing a chip (without looking into the bag and without revealing the choice to their partners) and playing the corresponding move, and the US Forces must choose without knowing the Insurgents' move. The US Forces should do whatever they think is best.
   b) Each student writes his/her move in the appropriate column on the 'Round 1' portion of the Section Guide.
   c) Players reveal moves to each other and note these in the appropriate 'Round 1' column. Use the matrix to determine payoffs for that play, and note these in the 'Round 1' table.
   d) The Randomizer (insurgent) ***must*** replace the chip after each draw and shake the bag a bit to mix the chips up. Replacement before resampling is critical.
5. Have students play approximately 15-20 very rapid draws. (You should designate a number of draws prior to playing. The table on the student Section Guide accommodates 18 plays. This stage should take approximately 5-8 minutes.) Students should tally their individual scores at the end of the designated number of draws.
6. **Interpretive Discussion 1.** Ask the following sequence of questions. Key points to elicit are in italics.
   a) What was the US Forces' experience in trying to respond to the Insurgents' moves? *You can't predict what the Insurgents are going to do, so it's very hard to determine what move to play. This differs from our previous games which did have equilibria in pure strategies.*
   b) Did some US Forces players score better than others? Why? *US Forces players whose Randomizers/Insurgents got bags from the ends of the probability distribution (strategies A and B above, or more generally, any strategy whose probabilities diverge from each other) should have higher scores than others.*
   c) Were you able to learn about the Insurgents' actions? Why? *The high probability of a particular move in some strategies (i.e., in strategies A and B the randomizer should have played one move frequently) makes the actor a bit more predictable, so selecting a 'best*

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[7] Again, color choices here are arbitrary but should be consistent across all responder bags. We recommend, if possible, that selected colors not duplicate those of the randomizers' bags. This facilitates interpretation.

response' is easier. US forces are thus able to adapt their strategies accordingly.[8]
c) What strategy is the 'best response' to a player who selects strategies randomly? *Because the US Forces could not predict with certainty the Insurgents' actions, the best the US forces could do is form some belief about the Insurgents' actions and select their strategy "randomly" based on that probability distribution (belief).*

7. Using the 'Round 1' portion of the instructor transparency on page 13 of this packet, or a similar chart drawn on the board, ask groups to share their scores. Compute the average score for each player in groups using the same distribution. (*Example:* Two groups played with Strategy A bags. One pair's Insurgents got 28 points, the other got 17. Record 22.5 in the table.) Discuss the results. Which groups did better, on average, than others? *Ideally, US Forces in the group playing the optimal strategy (strategy E above) will have the best scores. Insurgents in groups whose distribution of moves deviated substantially from the optimal should have done very poorly. (You will know which groups these are, but students won't. Draw attention to the scores, but do not reveal why this is the case.)* Leave the table on the board or projector for later comparison.
8. **Instructional Discussion.** Teach students to compute mixed strategies, using only the US Forces' side of the story. Begin by labeling the US Forces' Patrol A and Patrol B moves on the transparency as q and $1 - q$, respectively.

   US Forces want to make:

$$\text{EU (attack A)} = \text{EU (attack B)}$$
$$q(2) + (1 - q)(3) = q(4) + (1 - q)(1)$$
$$2q + 3 - 3q = 4q + 1 - q$$
$$-q + 3 = 3q + 1$$
$$2 = 4q$$
$$q = \tfrac{1}{2}, \text{ so } (1 - q) = \tfrac{1}{2}$$

   Give students a formal definition of mixed strategies: a mixed strategy for a game is a probability distribution on the set of its pure strategies (Morrow 1994: 82). Mixed strategy equilibrium exists when actors are indifferent among the choice of actions and randomize between action choices. If both players used mixed strategies, both make each other indifferent among their choices, producing equilibrium or steady-state.
9. Discuss the intuition behind the formal definition. Here, indifference and randomization are key concepts. Explain to students that the US Forces could maximize their payoffs by selecting strategies randomly using the proportions you just calculated. Why? Strategically, US Forces have incentives to play randomized mixed strategies in the face of uncertainty about the Insurgent's choices, to make the Insurgents indifferent among *their* choices. These proportions calculated above make the Insurgents ***indifferent*** between attacking area A and attacking area B because both moves have the same expected utility for the Insurgents when the US plays its moves with probabilities ½ and ½. When the Insurgents are indifferent and are effectively picking moves randomly, by coin toss or use of a spinner or other similar device, the US Forces have a chance of obtaining the outcome that makes them better off. When the US Forces choose randomly,

[8] Instructors in modeling classes may wish to introduce or highlight the notion of Bayesian updating here. See Rubenstein (1991); Reny and Robson (2001) discuss Bayesian interpretations of mixed strategies.

they ensure that the Insurgents cannot gain an advantage by knowing anything about the US Forces' likely moves. In discussion, contrast this with a strategy where the US Forces always protected A. Discuss how the insurgents could gain advantage under this strategy.

10. Distribute the reserved bags with the optimal Responder strategy to the US Forces players. Explain to them that the bag contains chips in proportions that match the optimal response strategy, and show them which colors in the US Forces bags correspond to which moves. Encourage them to make projections about what they think will happen to the scores when a) both actors select moves randomly, and b) US Forces play the optimal strategy. *Some advanced or intuitive students may realize that the effect of the US optimal strategy varies with the Insurgent strategy. US Forces facing Insurgents who play suboptimal strategies (i.e., who have bags with strategies A-D above) should have higher scores than US Forces facing Insurgents with the optimal strategy (strategy E above).*
11. **Round 2.** Inform students that in this round, both actors are to select their strategies randomly, by drawing a chip from the bag. (The Round 2 matrix includes color assignments for US Forces moves.) Both actors should draw before revealing their moves to their opponents. Scores should be recorded and tallied in the 'Round 2' table in the student Section Guide. Again, play a predesignated number of rounds (15-20 works best) in 5-8 minutes. The Section Guide accommodates 18 rounds. Play and discussion parallel Round 1 above.
12. **Interpretive Discussion 2.** Solicit reactions from students about what happened in the second round. How was it different from the first round? The latter question should be directed especially at students playing strategies A and B above.
13. Using the 'Round Two' part of the instructor transparency or a similar table drawn on the board, compute average scores for each player in each probability distribution. With the students, identify the 'winner' in each pair (who had the higher score, Insurgents or US Forces). How does this compare to the Round 1 scores and winners? Compare these results with their predictions from step 10 above. *In pairs where the Insurgents are playing the optimal strategy (strategy E above), the randomizer/Insurgent should do reasonably well even against the optimal response (US Forces) strategy. Non-optimal randomizer/Insurgent strategies, though, should do extremely poorly against the optimal response (US Forces) strategy.*
14. **Instructional Discussion 2.** Ask students to solve for the randomizer's (Insurgent's) optimal strategy using the class demonstration from step 8 as a guide. Remind them to label the strategies as p and 1-p. (The Section Guide and instructor transparency have room to write this above the move labels.) Have a student compute the solution ($p = 1/4$, $1 - p = 3/4$) on the board. For reference:

<u>Iraqi Insurgents want to make:</u>

EU (patrol A) = EU (patrol B)

$$p\,(4) + (1 - p)\,(2) = p\,(1) + (1 - p)(3)$$
$$4p + 2 - 2p = p + 3 - 3p$$
$$2p + 2 = -2p + 3$$
$$1 = 4p$$
$$p = 1/4, \text{ so } (1 - p) = 3/4$$

15. Explain to students that this is the optimal strategy for the Insurgents; Insurgents playing this strategy should have scored best in Round 1 and done well in Round 2. Have

randomizer/Insurgent students open their bags and count the chips to identify the probability distribution from which they were drawing their moves. (You may wish to note this in the data table.) Did the Insurgents playing the optimal strategy score according to expectations? *If the answer is no, be prepared to discuss reasons why this might have occurred.*

16. Write the complete equilibrium mixed strategy on the board as [(*A* = ¼, B = ¾), (*A* = ½, B = ½)] or in your preferred format. Discuss the general characteristics of mixed strategies[9]:
    a) Both players must play probabilistically. If one player's moves are predictable, even if only in sequence, then the other player has no need for randomization. *Good discussion examples here are the game Rock, Paper, Scissors (imagine what would happen if your opponent simply alternated between the three moves in some sequence), football play calling (what would happen if offenses simply alternated pass, rush, pass, rush?), etc.*
    b) All non-dominated strategies must be accounted for in the probability distribution, and the probabilities of all moves must sum to 1. *Explain to modeling classes that dominated strategies are effectively played with probability 0.*
    c) We solve for mixed strategies using <u>my</u> letters and <u>your</u> payoffs because I want to make you indifferent. *Students often are confused with which sets of letters and numbers go together. Remind them of the logic of the class exercise to help them remember.*
    d) All strategic-form games have an odd number of equilibria. Games with two equilibria in pure strategies, like Battle of the Sexes or Coordination, have an additional equilibrium in mixed strategies. *With modeling classes, discuss why this is true: mixed strategies provide an analytical tool for choosing or alternating between equilibria.*

<u>Enrichment, Reinforcement, and Assessment</u>

Pages 14-15 of this packet contain a set of exercises for solving for mixed strategies; solutions and additional examples begin on page 16. These examples are from comparative, international, and American politics. Instructors may choose to substitute any of these examples for the Iraqi Insurgents/US Forces example used above (with appropriate modifications to the student Section Guide, transparencies, and instructions).

This page is also appropriate for a homework assignment or groupwork discussion use in a modeling class. Exercises 5-7 are enrichment or 'Challenge' exercises which ask students to go beyond the set of steps identified here and demonstrated in the class plan; they may require skills beyond the prerequisite set identified at the start of this lesson plan. In these exercises, students solve for mixed strategies in 3x3 games. Both players have a dominated strategy in Exercise 5, meaning that the game reduces to a familiar 2x2. In Exercise 6, both have dominated strategies but the strategies must be eliminated sequentially. Finally, Exercise 7 remains a 3x3, though the zero payoffs cause many probability terms to drop out.

Advanced students will benefit from the Challenge exercises, though most if not all students should be able to complete Exercises 5 and 6. Advanced students can be encouraged to identify other examples of mixed strategies in political behavior and construct their own games and exercises. Additionally, you might ask advanced students to 'reverse-engineer' a mixed

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[9] This step may be omitted for introductory/exposure-level classes, depending on the instructor's pedagogical goals.

strategy solution to obtain a specified probability distribution (e.g., [0.5, 0.5]) without changing the pure strategy solutions or ordinal ranking of outcomes.

Assessment of most objectives is best accomplished through student responses to discussion questions and student observation during the activities. When used as a homework assignment, pages 13-14 serves as a more thorough assessment of individual mastery, particularly for the modeling class objective.

PS 160 Intro to World Politics
Section Guide: Patrolling In Iraq

	Iraqi Insurgents	
	Red	White
US Forces	Attack area A	Attack area B
Patrol area A	(4 , 2)	(2 , 4)
Patrol area B	(1 , 3)	(3 , 1)

I am playing as (circle): US FORCES IRAQI INSURGENTS

Round 1:

US move	Iraqi move	US score	Iraqi score

US move	Iraqi move	US score	Iraqi score

Round 1 scores: US total: _______ Iraqi total: _______

<u>Work and Notes Space</u>

Round 2:

	US Forces	Iraqi Insurgents Red Attack area A	 White Attack area B
Blue	Patrol area A	(4 , 2)	(2 , 4)
Green	Patrol area B	(1 , 3)	(3 , 1)

US move	Iraqi move	US score	Iraqi score

US move	Iraqi move	US score	Iraqi score

Round 1 scores: US total: _______ Iraqi total: _______

Work and Notes Space

Terms to Know
Mixed strategy
Indifference

ROUND 1

	Iraqi Insurgents *Red* Attack area A	*White* Attack area B
US Forces		
Patrol area A	(4 , 2)	(2 , 4)
Patrol area B	(1 , 3)	(3 , 1)

ROUND 2

		Iraqi Insurgents *Red* Attack area A	*White* Attack area B
	US Forces		
Blue	Patrol area A	(4 , 2)	(2 , 4)
Green	Patrol area B	(1 , 3)	(3 , 1)

	ROUND 1		ROUND 2	
	US Forces	Iraqi Insurgents	US Forces	Iraqi Insurgents
Strategy A				
Strategy B				
Strategy C				
Strategy D				
Strategy E				

Name______________________

PS 160 Intro to World Politics
Practicing Mixed Strategies in Politics

For each example below, solve the game for any solutions in pure strategies and any solution in mixed strategies. Show your work. Be sure to write the complete equilibrium/ia and circle it to indicate your 'final answer.'

1. The D-Day Invasion

Near at the end of the World War II, the Allied is preparing for the D-day invasion to gain a decisive victory. To stave off the Allied attack, Germany has to decide whether to defend Calais or Normandy. Calais is more valuable for both the Allied and Germany for strategic reasons. While the allies want to attack the place where the German defense is weak, Germany has to defend where it could be attacked.

		German Defense	
		Normandy	Calais
Allied Attack	Normandy	(50, -50)	(80, -80)
	Calais	(100, -100)	(40, -40)

2. Pushing through the Health Care Bill

After facing gridlock, Democrats and Republicans want to reach a bipartisan consensus, but each prefers its own bill to the opponents'.

		Democrats	
		Pass	Not Pass
Republicans	Pass	(1, 2)	(0, 0)
	Not Pass	(0, 0)	(2, 1)

Name______________________

3. Centralized Wage Bargaining
Labor relations often produce sharp political conflicts in industrial societies. The Employers' Association wants to avoid a strike (to avoid losing profit) but would rather reject the union wage when there is no threat of strike. The Union has to pay the cost of strike (forgoing salaries, delayed production, free-rider problems, etc.) but would rather strike if the association is likely to reject the proposed wage.

		Union	
		Strike	No Strike
Employers Association	Accept union wage	(1, 2)	(3, 3)
	Reject union wage	(0, 3)	(4, 1)

Solve games 4-7 on the back or on separate paper. Circle the complete equilibrium or equilibria as your 'final answer.'

4.

		Player 2	
		Left	Right
Player 1	Top	(2, 2)	(3, 1)
	Bottom	(3, 1)	(2, 2)

5. ***Challenge***

		Player 2		
		Left	Middle	Right
Player 1	Up	(10, 10)	(1, 20)	(0, 15)
	Center	(20, 1)	(5, 5)	(3, 2)
	Down	(15, 3)	(3, 4)	(5, 5)

6. ***Challenge***

		Player 2		
		L	M	R
Player 1	U	(9, 4)	(2, 1)	(8, 3)
	C	(7, 2)	(6, 7)	(1, 5)
	D	(5, 8)	(4, 6)	(3, 9)

7. ***Challenge***

		Player 2		
		L	M	R
Player 1	U	(5, 0)	(0, 5)	(6, 0)
	C	(1, 0)	(3, 0)	(0, 10)
	D	(7, 8)	(0, 0)	(0, 0)

Mixed Strategies Without Mixups
Solution Key for "Practicing Mixed Strategies in Politics"

To The Instructor:
The following examples, drawn from American, Comparative and International politics, can replace the Iraq game above as motivating examples for this lesson. Instructors can also utilize these examples for 1) solving mixed strategies as a math exercise, 2) discussing the discrepancy in the model and real politics, and 3) changing the payoffs to see how the strategic interactions change.

1. International Politics Example: D-Day Invasion
Near at the end of the World War II, the Allied is preparing for the D-day invasion to gain a decisive victory. To stave off the Allied attack, Germany has to decide whether to defend Calais or Normandy. Calais is more valuable for both the Allied and Germany for strategic reasons. While the allies want to attack the place where the German defense is weak, Germany has to defend where it could be attacked.

		German Defense	
		Normandy	Calais
Allied Attack	Normandy	(50, -50)	(80, -80)
	Calais	(100, -100)	(40, -40)

<u>Solution</u>: Note that this is a zero-sum game with no pure strategy equilibrium. Equilibrium mixed strategy is (2/3 Normandy, 1/3 Calais) for the allies and (4/9 Normandy, 5/9 Calais) for Germany.

<u>Additional IR Examples:</u> See Powner and Bennett, *Applying the Strategic Perspective (3rd ed., 2005)* Chapter 8, illustrating the strategic moves between Japan and the United States in the Pacific during the World War II. Bueno de Mesquita (2005, ch 8) also includes an example about sanctions.

2. American Politics Example: Pushing through the Health Care Bill
After facing gridlock, Democrats and Republicans want to reach a bipartisan consensus, but each prefers its own way. Republicans would rather not pass the health care bill while Democrats want the opposite.

		Democrats	
		Pass	Not Pass
Republicans	Pass	(1, 2)	(0, 0)
	Not Pass	(0, 0)	(2, 1)

<u>Solution</u>: (1/3 Pass, 2/3 Not Pass) for Republicans and (2/3 Pass, 1/3 Not Pass) for Democrats. Since this is one-shot game, the instructor can treat this situation as a large population case and interpret this situation where 1/3 Republican members vote for "Pass" and the rest for "Not Pass" (and the symmetric case for Democrats).

Additional American Politics Examples: Other examples can include 1) any congressional bills of different issues with conflicting objectives, 2) the relationship between legislative and bureaucrats vying for political power and discretionary capacity, and 3) local politics where conflicting opinions exist as to where to build public facilities (e.g. park, airport, or dam).

3. Comparative Politics Example: Wage Bargaining
Labor relations often produce sharp political conflicts in industrial societies.[10] The Employers Association wants to avoid a strike (not to lose a profit) but would rather reject the union wage when there is no threat of strike. The Union has to pay the cost of strike (forgoing salaries, delayed production, free-rider problems, etc.) but would rather strike if the association is likely to reject the proposed wage.

		Union	
		Strike	No Strike
Employers Association	Accept union wage	(1, 2)	(3, 3)
	Reject union wage	(0, 3)	(4, 1)

Solution: The Employers' Association has to adopt a mixed strategy of (2/3 Accept, 1/3 Reject) to make the Union indifferent between "Strike" and "Not Strike." In response, the Union takes the mixed strategy of (1/2 "Strike", 1/2 "Not Strike").

Additional Comparative Politics Examples: Other examples can include 1) an electoral competition where two candidates in an election proposing different platforms to attract voters, or 2) a political reform situation with reformers and conformists.

Solutions to Practice Games 4-7:

Solution for Game # 4
Player 1 plays (1/2 T, 1/2 B); likewise, Player 2 plays (1/2 L, 1/2 R)

Solution for Game # 5
The 3 x3 game is reduced to the following 2 x 2 game after eliminating dominated strategies (that is, U for Player 1 and L for Player 2).

		Player 2	
		M	R
Player 1	C	(5, 5)	(3, 2)
	D	(3, 4)	(5, 5)

Player 1 chooses a mixed strategy (1/4 C, 2/4 D); Player 2 chooses to play (1/2 M, 1/2 R)

[10] See Thelen (1994) for specific examples.

Solution for Game # 6
Like Game 5, Game 6 also reduces to a 2 x 2, but it requires students to eliminate dominated strategies sequentially: first Player 1's dominated strategy (B), and Player 2's (R). This results in the following game:

		Player 2	
		L	M
Player 1	U	(9, 4)	(2, 1)
	C	(7, 2)	(6, 7)

The mixed strategy equilibrium is for Player 1 to play (5/8 U, 3/8 M); Player 2 plays (2/3 L, 1/3 C).

Solution for Game # 7
Unlike Game 5 where dominated strategies are eliminated, no strategy is strictly is dominated in this case. For mixed strategy equilibrium, Player 1 should play (8/17 U, 4/17 C, 5/17 D); Player 2 should play (3/10 L, 6/10 M, 1/10 R)

References and Resources

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